Abstracts

Workshop on Operator Theory,
Complex Analysis and Applications

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Catalin Badea

Université de Lille I

badea@math.univ-lille1.fr

"Angle" between several subspaces and applications

A notion of "angle" between several closed subspaces of a Hilbert space is introduced, and some applications are presented. In particular, we show how operator theory bridges the gap between numerical analysis and Kazhdan's property (T). This is based upon joint work with Sophie Grivaux -Vladimir Muller and Yuri I. Lyubich.

Oscar Blasco

Universidad de Valencia

Oscar.Blasco@uv.es

Mixed norms, Kellog spaces and Toeplitz operators

This is joint work with S. Perez-Esteva (UNAM, Mexico). Let A_2 be the Bergman space in the unit disk $\mathbb{D}$. It is known that for a nonnegative $\varphi \in L^1(\mathbb{D})$, the Toeplitz operator

$$T_\varphi(f)(z) = \int_{\mathbb{D}} \frac{f(w)\varphi(w)}{(1-\overline{w}z)^2} dA(w) \tag{1}$$

belongs to the Schatten class S_p if and only if the Berezin transform $\tilde{\varphi} \in L^p(\mathbb{D}, d\lambda)$, where $d\lambda = \frac{dA}{(1-|z|^2)^2}$ and dA is the normalized Lebesgue measure on $\mathbb{D}$. The second author introduced the so called Schatten-Herz classes $S_{p,q}$ of all Toeplitz operators T_φ such that when decomposed as $T_\varphi = \sum_{n=1}^{\infty} T_{\varphi_n}$, with $\varphi_n = \mathcal{X}A_n\varphi$ and $A_n = \{z \in \mathbb{D},\, 1-2^{-n} \le |z| \le 1-2^{-n-1}\}$ they satisfy $\left(\sum_{n=1}^{\infty} \|T_{\varphi_n}\|_{S_p}^q\right)^{1/q} < \infty$, providing that for nonegative symbols $T_\varphi \in S_{p,q}$ is equivalent to the Berezin transform $\widetilde{\varphi}$ belonging to the Herz space $\mathcal{K}_q^{p,-2/p}$.

In this talk we consider classes of compact operators T on A_2 with Berezin transform $\widetilde{T}$ belonging to the mixed norm spaces $L^{p,q,\alpha}$ defined by the condition

$$\|f\|_{L^{p,q,\alpha}} = \left(\int_0^1 (1-r)^{q\alpha-1}(\int_0^{2\pi} |f(re^{i\theta}|^p \frac{d\theta}{2\pi})^{q/p} dr)\right)^{1/q} < \infty.$$

This question turns out to be related with the Kellog space $\ell(p,q)$, consisting of complex sequences (λ_n), such that

$$\|(\lambda_n)\|_{\ell(p,q)} = \left(\sum_{k\ge 0}(\sum_{n\in I_k} |\lambda_n|^p)^{q/p}\right)^{1/q} < \infty,$$

where $I_k = [2^k-1, 2^{k+1}) \cap (\mathbb{N}\cup\{0\})$ for $k \in \mathbb{N}\cup\{0\}$ and $0 < p,q < \infty$.

Defining the class $K(p,q)$ of all positive compact operators T such that the sequence $(Te_n, e_n)_n \in \ell(p,q)$ we show the relation between $K(p,q)$ and the norm $\|(1-|z|)^{-2/p}\widetilde{T}\|_{L^{p,q,1/p}}$, and study Toeplitz operators T_φ with $\varphi \ge 0$ and such that Berezin transform $\widetilde{\varphi} \in L^{p,q,\alpha}$ showing that $\widetilde{\varphi} \in L^{p,q,\alpha}$ if and only $T_{(1-|z|)^a\varphi} \in S_{p,q}$ giving a new characterization of the Schatten-Herz Toeplitz operators.

Marcus Carlsson

Purdue University, West Lafayette, USA

mcarlsso@math.purdue.edu

Finite interval convolution operators and a Beurling's theorem for $L^2((-1,1))$?

Let $\Phi : (-1,1) \to \mathbb{C}$ be a function and consider the operator $\Gamma_\Phi : L^2((0,1)) \to L^2((0,1))$ given by

$$\Gamma_\Phi(F)(x) = \int_0^1 \Phi(x-y)F(y)dy, \quad 0 < x < 1.$$

It goes under names like “finite interval convolution operator”, “Truncated Wiener-Hopf operator” “Toeplitz operator on the Paley Wiener space” or “Truncated Hankel operator on $\mathbb{R}$”. We consider norm estimates and show that it shares many properties with Hankel operators, in particular we provide a Nehari type norm characterization, as well as norm and compactness characterization in terms of the sequential BMO-space.

Questions concerning relationship between kernel and symbol has led me to a conjecture which could be called “a finite interval version of Beurlings theorem”. In joint work with C. Sundberg, we can prove this for a dense subset, but in general the it is open. I will present the conjecture and our partial results.

Luís Castro

Universidade de Aveiro

castro@ua.pt

A transmission problem with imperfect contact on unbounded double periodic multiply connected domains

An analysis of the flux of certain unbounded doubly periodic multiply connected domains with circle disjoint components is performed. This is done under generalized non-ideal contact conditions on the boundary between domain components which include analytic given data. A formula for the flux which depends on the conductivity of components, their radii, centers, the conductivity of the matrix, and also certain values of special Eisenstein functions is derived. Existence and uniqueness of solution to the problem is obtained by using a transmission problem with imperfect contact for analytic functions in corresponding Hardy spaces. The talk is based on a joint work with E. Pesetskaya.

John B. Conway

George Washington University

conway@gwu.edu

Powers and direct sums

If A is an operator on a Hilbert space, when is A^2 similar to $A \oplus A$? This talk explores this question, obtains some results, and poses some questions.

Omar El-Fallah

Université Mohammed V, Rabat

elfallah@fsr.ac.ma

Level sets and composition operators on the Dirichlet space

We denote by $\mathcal{D}$ the classical Dirichlet space. This is the space of all analytic functions f on $\mathbb{D}$ such that

$$\mathcal{D}(f) := \int_{\mathbb{D}} |f'(z)|^2 \, dA(z) < \infty,$$

where $dA(z)$ stands for the normalized area measure in $\mathbb{D}$. Let φ be a holomorphic self-map of $\mathbb{D}$. The composition operator C_φ on $\mathcal{D}$ is defined by

$$C_\varphi(f) = f \circ \varphi, \qquad f \in \mathcal{D}.$$

We are interested herein in describing the spectral properties of the composition operator C_φ, such as compactness and Hilbert-Schmidt class membership, in terms of the size of the level sets, $E_\varphi(s) = \{\zeta \in \mathbb{T}\colon\ |\varphi(\zeta)| \geq s\}$, of φ.

Alexei Karlovich
Universidade Nova de Lisboa
oyk@fct.unl.pt

Singular integral operators with slowly oscillating coefficients and slowly oscillating non-Carleman shifts

Suppose α is an orientation preserving diffeomorphism (shift) of $R_+ = (0, \infty)$ onto itself with the only fixed points 0 and ∞. We establish a Fredholm criterion for the singular integral operator

$$(aI - bW_\alpha)P_+ + (cI - dW_\alpha)P_-$$

acting on $L^p(R_+)$ with $1 < p < \infty$, where $P_\pm = (I \pm S)/2$, S is the Cauchy singular integral operator, and $W_\alpha f = f \circ \alpha$ is the shift operator, under the assumptions that the coefficients a, b, c, d and the derivative α' of the shift are bounded and continuous on R_+ and may admit discontinuities of slowly oscillating type at 0 and ∞. This is a joint work with Amarino Lebre and Yuri Karlovich.

Christian Le Merdy
Université de Franche-comté, Besanon
christian.lemerdy@univ-fcomte.fr

Square function estimates for analytic operators and applications

Let $T\colon L^p(\Omega) \to L^p(\Omega)$ be a positive contraction. Assume that T is 'analytic', that is, $\sup_{n\geq 1} n\|T^n - T^{n-1}\| < \infty$. We show that T satisfies various square function estimates, a typical one being the following: There exists a constant $C > 0$ such that

$$\Big\| \Big(\sum_{n=1}^{\infty} n \big| T^n(x) - T^{n-1}(x) \big|^2 \Big)^{\frac{1}{2}} \Big\|_p \leq C \|x\|_p$$

for any $x \in L^p(\Omega)$. Such estimates are closely related to maximal ergodic inequalities and to functional calculus problems.

Maria Teresa Malheiro
Universidade do Minho
mtm@math.uminho.pt

Factorization on a torus and Riemann-Hilbert problems

A factorization, which is shown to be a generalization of Wiener-Hopf factorization, is studied for Hölder continuous functions defined on a contour Γ that is the pullback of $\dot{\mathbb{R}}$ (or the unit circle) in a Riemann surface Σ of genus 1. The existence of a holomorphic Σ-factorization for every invertible function in that class is established and formulas are given for the factors. A new concept of meromorphic Σ-factorization is introduced and studied, and its relation with holomorphic Σ-factorization is discussed. This is applied to study and solve some vectorial Riemann Hilbert problems, including Wiener-Hopf matrix factorization, as well as to study some properties of Toeplitz operators with 2×2 matrix symbols.

This is based on a joint work with M. C. Câmara.

Mostafa Mbekhta
Université de Lille I
Mostafa.Mbekhta@math.univ-lille1.fr

On linear map preserving generalized invertibility

Let H be an infinite-dimensional complex separable Hilbert space and $\mathcal{B}(H)$ the algebra of all bounded linear operators on H. In this talk, we discuss the following new results:

Theorem I Let H be an infinite-dimensional separable Hilbert space and $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ a linear map preserving generalized invertibility in both directions. Assume that ϕ is surjective up to finite rank operators. Then

$$\phi(\mathcal{K}(H)) \subseteq \mathcal{K}(H)$$

and there exist an invertible element a $\in \mathcal{C}(H)$ and either an automorphism $\tau : \mathcal{C}(H) \to \mathcal{C}(H)$ or an anti-automorphism $\tau : \mathcal{C}(H) \to \mathcal{C}(H)$ such that the induced map $\varphi : \mathcal{C}(H) \to \mathcal{C}(H)$, $\varphi(A + \mathcal{K}(H)) = \phi(A) + \mathcal{K}(H)$, $A \in \mathcal{B}(H)$, is of the form

$$\varphi(\mathrm{x}) = \mathrm{a}\tau(\mathrm{x})\,, \qquad \mathrm{x} \in \mathcal{C}(H).$$

Theorem II Let H be an infinite-dimensional separable Hilbert space and $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ a linear map preserving semi-Fredholm operators in both directions. Assume that ϕ is surjective up to compact operators. Then

$$\phi(\mathcal{K}(H)) \subseteq \mathcal{K}(H)$$

and the induced map $\phi : \mathcal{C}(H) \to \mathcal{C}(H)$ is either an automorphism, or an anti-automorphism multiplied by an invertible element $\mathrm{a} \in \mathcal{C}(H)$.
Theorem III Under the same hypothesis and notation as in (II), the following statements hold true:
(i) ϕ preserves Fredholm operators in both directions;
(ii) there is an $n \in \underline{\mathrm{Z}}$ such that either

$$\mathrm{ind}(\phi(T)) = n + \mathrm{ind}(T) \quad \text{or} \quad \mathrm{ind}(\phi(T)) = n - \mathrm{ind}(T)$$

for every Fredholm operator T.

Observe that, every $n \times n$ complex matrix has a generalized inverse (resp. is semi-Fredholm, Fredholm), and therefore, every linear map on a matrix algebra preserves generalized invertibility (resp. semi-Fredholm, Fredholm) in both directions. So, we have here an example of a linear preserver problem which makes sense only in the infinite-dimensional case.

We shall say that ϕ *preserves strongly generalized invertibility* if $\phi(y)$ is a generalized inverse of $\phi(x)$ whenever y is a generalized inverse of x.

Theorem IV Let A and B be unital complex Banach algebras and let $\phi : A \to B$ be a unital additive map. Then ϕ preserves strongly generalized inverses if and only if ϕ is a Jordan homomorphism.

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Lina Oliveira
Instituto Superior Técnico, Lisbon
linaoliv@math.ist.utl.pt

On Lie ideals of nest algebras

The structure of Lie ideals of nest algebras has been investigated by many authors for at least a decade. Two main lines of this research might be essentially described as focusing either on the connection between Lie ideals and associative ideals or on similarity invariant subspaces. In this talk we present a different approach based on the set of finite rank operators in the Lie ideal. As a starting point we show that the finite rank operators lying in a norm closed Lie ideal of a continuous nest algebra are decomposable and then proceed to obtain the complete characterisation of these operators. The continuity of the nest is essential to obtain these results.

Jonathan R. Partington

University of Leeds

J.R.Partington@leeds.ac.uk

Inner functions, restricted shifts and interpolation

Motivated by the study of restricted shift operators and their numerical ranges, we consider certain interpolation problems on the unit circle involving inner functions. This is joint work with I. Chalendar and P. Gorkin.

Leiba Rodman

William and Mary College, Williamsburg

lxrodm@math.wm.edu

Factorization vs invertibility

It is well known that for matrix functions on the unit circle in the Wiener class invertibility is equivalent to factorability (in the sense of Wiener-Hopf factorization). This is no longer generally true for matrix functions on compact abelian groups other than the unit circle. Moreover, if the dual abelian group contains a copy of $\mathbf{Z}^3$, then the set of invertible matrix functions (again, in the Wiener class) is not dense in the set of factorable matrix functions, a result obtained recently jointly with Brudnyi and Spitkovsky. In the talk various connections and disconnections between invertibility and factorability will be explored, and open problems stated.

Pedro A. Santos

Instituto Superior Técnico, Lisbon

pasantos@math.ist.utl.pt

Banach algebra techniques for spline approximation methods

We will discuss some issues that appear when applying Banach algebra techniques to study stability of spline approximation methods for convolution operators acting on $L_p(\mathbb{R})$.

Ilya Spitkovsky
William and Mary College, Williamsburg
ilya@math.wm.edu

Kernels of asymmetric Toeplitz operators, and related almost periodic factorizations

We will introduce the notion of asymmetric Toeplitz operators and show how the description of their kernels can be used in the construction of (not necessarily canonical) factorization of almost periodic matrix functions. This approach yields a different view at some known cases (such as one sided, big gap, and binomial situations), as well as allows to discover new factorable classes.

The talk is based on joint work with M. C. Camara and Yu. I. Karlovich.

Elizabeth Strouse
Université Bordeaux I
Elizabeth.Strouse@math.u-bordeaux1.fr

Some simple Toeplitz operators on the Bergman space

There are many classical and beautiful results about Toeplitz operators on the Hardy space which can be explained simply in terms of their matrix representation. When one begins to study Toeplitz operators on the Bergman space, one sees that, for one reason or another that, in this context, things are usually much more complicated and difficult to understand. I will discuss a set of simple examples that in one way or another explain many of these differences.

Franciszek Szafraniec
Institute of Mathematics, Jagiellonian University, Kraków
umszafra@cyf-kr.edu.pl

An attempt at defining Hankel operators in unbounded domains

Bearing complexity of Hankel operators in typical unbounded domains in mind I intend to propose an axiomatic in a sense approach to their definitions. It is somehow in spirit of V. Pták and P. Vrbovaá, Operators of Toeplitz and Hankel type, *Acta Math. Sci.*(*Szeged*) 52(1988), 117-140 though more analytical in the nature. What differs this from the aforesaid approach is the target space of the would-be Hankel operator to be constructed first. Then the essential problem turns out how to make both spaces integrable. A substantial part of the talk develops this issue as well.

Estelita Vaz
Universidade do Minho
evaz@mct.uminho.pt

An application of the Riemann-Hilbert problem in General Relativity

Due to the complexity on solving the Einstein field equations, geometric assumptions are often considered. Spherical symmetry and axial symmetry are examples corresponding to interesting matter distributions. Here the Riemann Hilbert problem is used to obtain solutions of the Einstein equations for a mater distribution possessing symmetry properties.

Hassan Zerouali
Université Mohammed V, Rabat
zerouali@fsr.ac.ma

Spectral theory invariance under operator equations

TBA
